Student Goals – Math 3283

Mathematical Practice

Proficient students expect mathematics to make sense. They take an active stance in solving mathematical problems. When faced with a non-routine problem, they have the courage to plunge in and try something, and they have the procedural and conceptual tools to carry through. They are experimenters and inventors, and can adapt known strategies to new problems. They think strategically.

Students who engage in these practices discover ideas and gain insights that spur them to pursue mathematics beyond the classroom walls. They learn that effort counts in mathematical achievement.[a] These are practices that expert mathematical thinkers encourage in apprentices. Encouraging these practices in our students should be as much a goal of the mathematics curriculum as is teaching specific content topics and procedures.[b] Taken together with the Standards for Mathematical Content, they support productive entry into college courses or career pathways.

Core Practices - Students can and do:

1. Attend to precision.
   Mathematically proficient students organize their own ideas in a way that can be communicated precisely to others, and they analyze and evaluate others' mathematical thinking and strategies noting the assumptions made. They clarify definitions. They state the meaning of the symbols they choose, are careful about specifying units of measure and labeling axes, and express their answers with an appropriate degree of precision. Rather than saying, "let v be speed and let t be time," they would say "let v be the speed in meters per second and let t be the elapsed time in seconds from a given starting time." They recognize that when someone says the population of the United States in June 2008 was 304,059,724, the last few digits indicate unwarranted precision.

2. Construct viable arguments.
   Mathematically proficient students understand and use stated assumptions, definitions and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They break things down into cases and can recognize and use counterexamples. They use logic to justify their conclusions, communicate them to others and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose.

3. Make sense of complex problems and persevere in solving them.
   Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They consider analogous problems, try special cases and work on simpler forms. They evaluate their progress and change course if necessary. They try putting algebraic expressions into different forms or try changing the viewing window on their calculator to get the information they need. They look for correspondences between equations, verbal descriptions, tables, and graphs. They draw diagrams of relationships, graph data, search for regularity and trends, and construct mathematical models. They check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?"

4. Look for and make use of structure.
   Mathematically proficient students look closely to discern a pattern. For example, in $x^2 + 5x + 6$ they can see the 5 as $2 + 3$ and the 6 as 2×3. They recognize the significance of an existing line in a geometric figure and can add an auxiliary line to make the solution of a problem clear. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects. For example, by seeing $5 – 3(x – y)^2$ as 5 minus a positive number times a square, they see that it cannot be more than 5 for any real numbers x and y.[b]

5. Look for and express regularity in repeated reasoning.
   Mathematically proficient students pay attention to repeated calculations as they carry them out, and look both for general algorithms and for shortcuts. For example, by paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, they might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel in the expansions of $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ leads to the general formula for the sum of a geometric series. As they work through the solution to a problem, proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.[b]

6. Make strategic decisions about the use of technological tools.
   Mathematically proficient students consider the available tools when solving a mathematical problem, whether pencil and paper, ruler, protractor, graphing calculator, spreadsheet, computer algebra system, statistical package, or dynamic geometry software. They are familiar enough with all of these tools to make sound decisions about when each might be helpful. They use mathematical understanding and estimation strategically, attending to levels of precision, to ensure appropriate levels of approximation and to detect possible errors. They are able to use these tools to explore and deepen their understanding of concepts.

(a) For the importance of students' beliefs about effort, see the National Mathematics Advisory Panel's Report of the Task Group on Learning Processes, p. 4-10 (2008). (b) Cuoco, A., Goldenberg, E. P., and Mark, J., *Journal of Mathematical Behavior*, 15 (4), 375-402, 1996; *Focus in High School Mathematics*. Reston, VA: NCTM, in press; Harel, G., What is Mathematics? A Pedagogical Answer to a Philosophical Question. In R. B. Gold & R. Simons (Eds.), *Current Issues in the Philosophy of Mathematics From the Perspective of Mathematicians*, Mathematical Association of America, 2008.